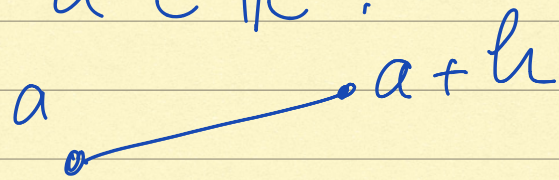
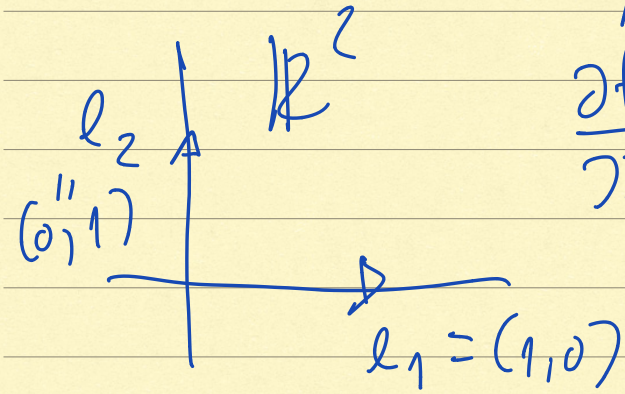
$\boxed{\text{REVISÃO}}$ 6/10/2020

$f: \mathbb{R}^n \longrightarrow \mathbb{R}^m$, diferenciável em $a \in \mathbb{R}^n$:



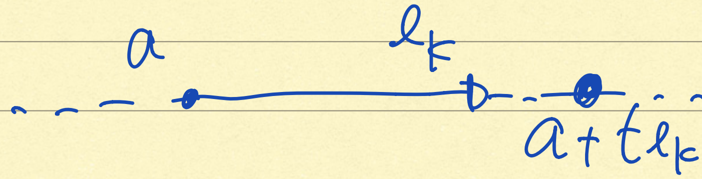
$$f(a+h) - f(a) - Df(a)h = o(h)$$

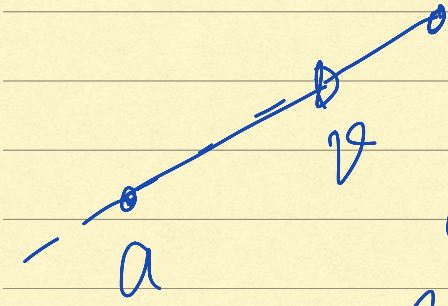
$\underset{m \times n}{Df(a)} \longleftrightarrow$ derivadas parciais.



$$\frac{\partial f}{\partial x}(a,b) = \lim_{t \to 0} \frac{f(a+t,b) - f(a,b)}{t}$$

$$\frac{\partial f}{\partial y}(a,b) = \dots$$





derivada de f segundo o vector v.

$$\boxed{D_v f(a) = \lim_{t \to 0} \frac{f(a+tv)-f(a)}{t}}$$

definição

$$\equiv \frac{\partial f}{\partial v}(a)$$

teorema

Se f for diferenciável em a

então, $\boxed{\frac{\partial f}{\partial v}(a) = Df(a)v}$

Quais são as funções diferenciáveis

$\boxed{\text{1- funções LINEARES}}$

$$f : \mathbb{R}^n \to \mathbb{R}^m \text{, linear}$$

$$f(x) = \boxed{A}\,x \quad , \; A_{m \times n}$$

$$f(a+h) = A(a+h) = \underbrace{Aa}_{f(a)} + Ah$$

$$f(a+h) = f(a) + Ah$$

$$f(a+h) - f(a) - \underset{\substack{\| \\ \boxed{Df(a)}}}{\boxed{A}}\,h = \underset{\substack{\equiv \\ o(h)}}{0}$$

$$\boxed{Df(a) = A}$$

Exemplo:

linear

$$f(x,y) = x = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$Df(x,y) = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\frac{\partial f}{\partial x}(x,y) = 1 \quad ; \quad \frac{\partial f}{\partial y}(x,y) = 0$$

CDI-I:

$$f(x) = ax$$

$$f'(x) = \underline{a}$$

2 - Funções constantes

$$f: \mathbb{R}^n \to \mathbb{R}^m, \quad f(x) = c$$

constante.

$$f(a+h) = c \quad ; \quad f(a) = c$$

$$f(a+h) - f(a) = 0$$

$$f(a+h) - f(a) - \underbrace{\boxed{0}}_{Df(a)} h = \underset{o(h)}{\underset{|||}{0}}$$

$$Df(a) = 0 \quad (\text{matriz nula})$$

Ex: $f(x,y) = \underbrace{1}_{\text{Constante}} + \underbrace{x + 2y}_{\text{linear}}$

$$\boxed{\text{3 - Soma de diferenciáveis}}$$

$f, g : \mathbb{R}^n \longrightarrow \mathbb{R}^m$, dif. em $\underline{a}$

$$(f+g)(a+h) = f(a+h) + g(a+h)$$

$$(f+g)(a) = f(a) + g(a)$$

$$\boxed{\begin{array}{l} f(a+h) - f(a) - Df(a)\,h = o_f(h) \\ g(a+h) - g(a) - Dg(a)\,h = o_g(h) \end{array}}$$

+ Somar

$$\underbrace{f(a+h)+g(a+h)}-\left(f(a)+g(a)\right)-$$

$$-\underbrace{\left(Df(a)+Dg(a)\right)}_{\text{derivada}}h=\underset{f}{o}(h)+\underset{g}{o}(h)$$

$$D(f+g)(a)=Df(a)+Dg(a)$$

$$\|$$

CDI-I

$$(f+g)'(a)=f'(a)+g'(a)$$

$$\|$$

$\boxed{\text{Caso importante:}}$

$$f(x,y)=xy$$

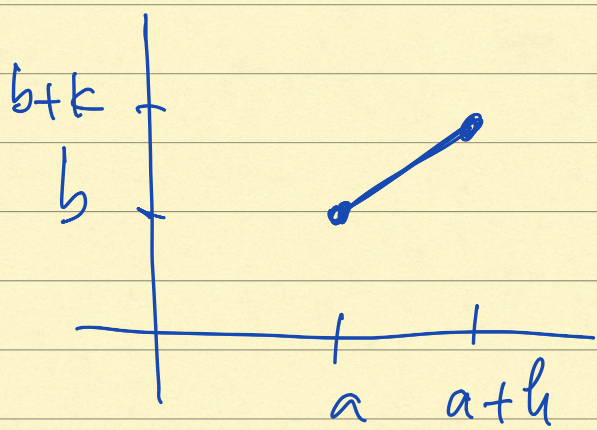
$f:\mathbb{R}^2 \longrightarrow \mathbb{R}$, $f(x,y)=xy$

$$f(x,y) = \underbrace{g(x,y)}_{x} \times \underbrace{h(x,y)}_{y}$$

$\boxed{f(x,y)=xy}$ é dif. em (a,b)?

usar a definição:

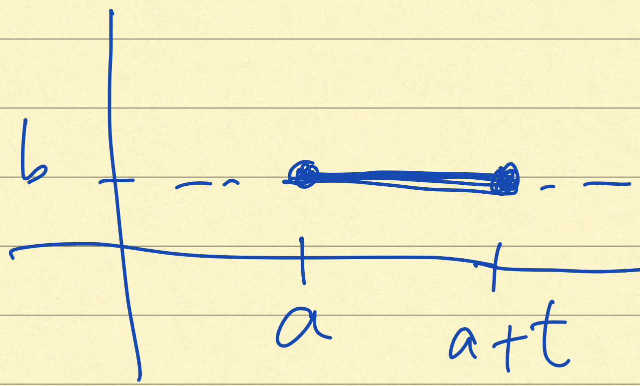
$$f(a+h,b+k)-f(a,b)-\underbrace{Df(a,b)(h,k)}\overset{?}{=}o(h,k)$$



$$(a+h)\times(b+k)-ab-bh-ak \overset{?}{=} o(h,k)$$

$$\cancel{ab}+\cancel{ak}+\cancel{bh}+hk\cancel{-ab}\cancel{-bh}\cancel{-ak}=hk$$

$$\frac{\partial f}{\partial x}(a,b) = \lim_{t \to 0} \frac{f(a+t,b) - f(a,b)}{t}$$



$$f(x,y) = xy$$

$$\frac{\partial f}{\partial x}(a,b) = \lim_{t \to 0} \frac{(a+t)b - ab}{t} = b$$

$$\frac{\partial f}{\partial y}(a,b) = \lim_{t \to 0} \frac{f(a,b+t) - f(a,b)}{t} = \underline{\underline{a}}$$

$$hk \overset{?}{=} o(h,k)$$

$$\lim_{(h,k)\to(0,0)} \frac{hk}{\|(h,k)\|} \overset{?}{=} 0$$

$$\left|\frac{hk}{\|(h,k)\|}\right| = \left|\frac{hk}{\sqrt{h^2+k^2}}\right|$$

!!!

$$= \frac{|h|\,|k|}{\sqrt{h^2+k^2}} \le |k| \to 0$$

≤ 1

$\therefore$ $f(x,y) = xy$ é dif. em (a,b)

$$Df(a,b) = \begin{bmatrix} b & a \end{bmatrix}.$$

$$D(f\cdot g)(a) = f(a)Dg(a) + g(a)Df(a)$$

$$(fg)' = fg' + gf'$$

Exemplo: $f(x,y) = \underbrace{x+y}_{\text{linear}} + 2xy$

f é soma de diferenciáveis

$\Rightarrow$ f é diferenciável.

$Df(a,b) = \begin{bmatrix} 1+2b & 1+2a \end{bmatrix}.$